

Reply to Comment on “Simple One-Dimensional Model of Heat Conduction which Obeys Fourier’s Law”

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In this reply we answer a comment by A. Dhar on our Letter [1]. In that paper we studied thermal conductivity in a one-dimensional gas of hard point alternating-masses particles, demonstrating that Fourier’s law holds both in its statical and dynamical aspects.

Dhar’s main point is about the asymptotic behavior of the total energy current self correlation function $C(t)$. He fits $C(t) \sim t^{-0.83}$, implying an infinite thermal conductivity, while we obtained $C(t) \sim t^{-1.3}$ and thus a finite κ . In our opinion, Dhar’s result is a consequence of autocorrelations due to finite size effects. In order to show this point, we plot in fig. 1.a $C(t)$ for a system size $N = 1000$. Here we can study two different regions: (1) one for $\ln(t/t_0) \in [8, 9]$, and (2) other with $\ln(t/t_0) > 10$. A power law fit to the first region yields $C(t) \sim t^{-1.3}$, while a fit to the second region yields $C(t) \sim t^{-0.88}$, very similar to Dhar’s result. In fact, similar behavior has been measured by Savin et al[2]. However, we think that only region (1) corresponds to the infinite system asymptotic behavior. As an example of the previous statement, we can study the asymptotic behavior of the local energy current self correlation function $c(t)$ for the equal masses gas. Following Jepsen [3], it can be shown analitically, after a lengthy calculation, that

$c(t) \sim t^{-3}$ for this system. We have measured $c(t)$ [1] for equal masses in a finite system in the *canonical ensemble* (not in the zero-momentum ensemble, contrary to Dhar’s comment). Fig. 1.b shows $c(t)$ for $N = 500$. It is remarkable that we can also define here two different regions: (1) one for $\ln(t/t_0) \in [5.1, 5.8]$, where a power law fit yields $|c(t)| \sim t^{-3}$, and (2) one for $\ln(t/t_0) > 6$, where a power law fit yields $|c(t)| \sim t^{-0.83}$. We recover the theoretically predicted asymptotic bulk behavior in region (1), while region (2) should be due to finite size effects. Moreover, it is intriguing that the finite size time decay exponent (~ 0.83) is almost the same both in the different masses case and the equal masses one. This fact points out the existence of an underlying common finite size mechanism, responsible of this spureous long time decay. In conclusion, coming back to the different masses case, we think that the above example indicates that only region (1) of fig. 1.a represents the asymptotic bulk behavior. Hence, any conclusion about system’s conductivity derived from region (2) should be misleading.

Let’s clarify now some other minor points raised in Dhar’s comment. First, we *do not* observe linear temperature profiles. They are linear in the central region and curved near the boundaries, which is consistent with the finding of finite conductivity.[1] Second, the validity of our deterministic heat bath and our local temperature measure has been carefully tested. On the other hand, it can be shown that $C(t) \sim Nc(t) + \sum_{i \neq l} c_{i,l}(t)$. Hence, for *regular* systems, where non-local time correlation functions $c_{i,l}(t)$ decay fast enough with distance, one expects a similar long time decay for both $C(t)$ and $c(t)$. However, there are *anomalous* systems, as the (non-ergodic) equal masses gas, for which $c_{i,l}(t)$ decays very slowly, or does not decay at all, and thus $C(t)$ and $c(t)$ behave completely different.

In conclusion, we firmly confirm, after a global, consistent analysis of the problem, our previous results [1], i.e. that our one-dimensional system has a finite thermal conductivity in the Thermodynamic Limit, thus obeying Fourier’s law.

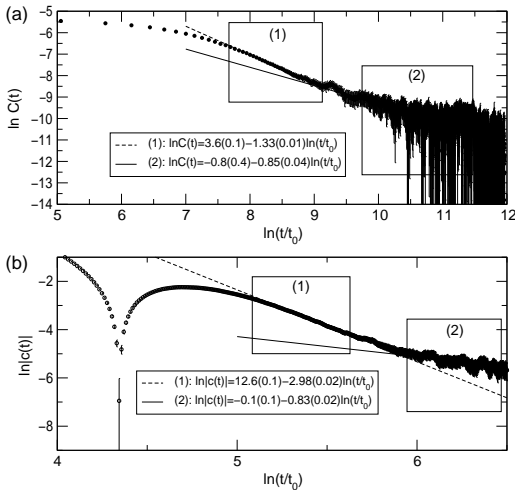


FIG. 1: (a) $C(t)$ for the different masses system with $N = 1000$. (b) $c(t)$ for the equal masses system with $N = 500$. The insets show the results of a power law fit for both regions (1) and (2). t_0 is the mean collision time.

- [1] P.L. Garrido et al, Phys. Rev. Lett. **86**, 5486 (2001).
- [2] A.V. Savin et al, Phys. Rev. Lett **88**, 154301-1 (2002).
- [3] D.W. Jepsen, J. Math. Phys. (N.Y.) **6**, 405 (1965).